Problems 12/29/2025
Solutions to the problems below will be published today

MathLovers

Problem 1. Let $a, b, c > 0$ and $a + b + c = 67$. Prove that

$$4\left(a\sqrt[3]{a} + b\sqrt[3]{b} + c\sqrt[3]{c}\right) + a + b + c \geqslant 1000.$$

Problem 2. Define the sequence a_n by: $a_1 = 1, a_{n+1} = a_n + n + 1$ for every positive integer n. A positive integer k is called *Chinese-friendly* if the number $8k + 1$ is a perfect square and

$$\prod_{i=1,\ a_i \leqslant k} a_i$$

is a perfect square. Prove that there are infinitely many Chinese-friendly numbers.

Good luck!

With this set of problems, we close not only the year 2025 but also the series of problems regularly published by our project.

Thank you all for your participation, time, and mathematical engagement.

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