Problem 1. On a rectangular baking tray, $n \in \mathbb{N}$ gingerbreads fit. Show that on the same tray, $4n$ gingerbreads of half the diameter can fit.

Problem Author: Grzegorz Rudnicki
Solution: Notice that any rectangle can be divided into 4 rectangles similar to it at a scale of $\frac{1}{2}$. Therefore, we can divide the baking tray into 4 smaller trays. Consider a dilation at a corner of the tray with scale $\frac{1}{2}$. The gingerbreads on the tray map to gingerbreads half the size, placed on one of the smaller copies of the tray. We can do the same for the remaining corners. Since each smaller tray can fit n smaller gingerbreads, the entire tray can fit $4n$. ∎

Problem 2. Santa Claus has a spherical sack of radius R. Since Santa is worried about Polish students' reluctance to study mathematics, all children will receive textbooks in this wonderful subject this year. The books have a very practical square shape. What is the maximum possible size (surface area of the cover) of a textbook so that it fits in the sack? Assume that the book is infinitely thin.

Problem Author: Grzegorz Rudnicki
Solution: Consider a cross-section of the sack through the plane containing the book. It is a circle of radius R. Let the dimensions of the book be $2a \times 2b$. The area of the book is $4ab$. If a rectangle is inscribed in a circle, its diagonal is the diameter of the circle. Thus, the condition

$$4a^2 + 4b^2 = 4R^2 \quad \text{or equivalently} \quad a^2 + b^2 = R^2$$

must hold. By the AM-GM inequality, we have

$$ab \leqslant \frac{a^2 + b^2}{2} = \frac{R^2}{2}.$$

Hence, the maximal possible area is $\frac{R^2}{2}$. This value is achieved for a square with side $R\frac{\sqrt{2}}{2}$. ■