Problems 12/01/2025

The solutions to the problems below
will be published on Thursday, 12/04/2025

Problem 1. Prove that there exists $A \in \mathbb{R}$ such that for every $n \in \mathbb{N}$ we have

$$\left(1+\frac{1}{n}\right)^n \leqslant A.$$

Problem 2. Let O be a point inside an acute triangle ABC. Let p, q be distinct lines passing through O, both intersecting sides AB and AC. Denote $K = p \cap AB$, $L = p \cap AC$, $M = q \cap AC$, $N = q \cap AB$. The circumcircles of triangles NKO and MOL intersect at $P \neq O$. Assume that the points A, N, K, B and A, L, M, C lie on sides AB and AC respectively in this order. Prove that

$$\sphericalangle BAC = \sphericalangle PKL + \sphericalangle PMN.$$

Good luck!

We encourage you to submit your solutions via the website
https://mathlovers.eu/submit-solution/!