Problem 1. Prove that every integer can be represented in the form $\pm 1^2 \pm 2^2 \pm \ldots \pm k^2$ for some $k > 1$ and some choice of signs.

Author: Robert Rośczak (popular problem)
Solution: Notice that $(a-1)^2 - a^2 - (a+1)^2 + (a+2)^2 = 4$. It remains to construct the residues $0, 1, 2, 3$:

- $1^1 = 1 \equiv 1 \pmod 4$,
- $-1^2 + 2^2 - 3^2 = -6 \equiv 2 \pmod 4$,
- $-1^2 \equiv 3 \pmod 4$,
- $\emptyset \equiv 0 \pmod 4$.

Problem 2. For sets X, Y of integers we define $X - Y = \{x - y \mid x \in X, y \in Y\}$. Given are sets A, B, C of integers. Prove the inequality

$$|A - B| \cdot |A - C| \geqslant |A| \cdot |B - C|.$$

Source: Ruzsas Triangle Inequality
Choice: Robert Rościak
Solution: For each element $i \in B - C$ fix uniquely b_i, c_i satisfying $b_i \in B, c_i \in C, b_i - c_i = i$. If there are many such pairs then choose one.

We will prove that the function $\varphi : (a, n) \mapsto (a - b_n, a - c_n)$ is injective. Note that the function $\varphi' : (x, y) \mapsto (x + b_{x-y}, x - y)$ allows one to recover the argument of φ from its value. This means that to each value there corresponds at most one argument, hence the function is injective, from which the inequality follows.