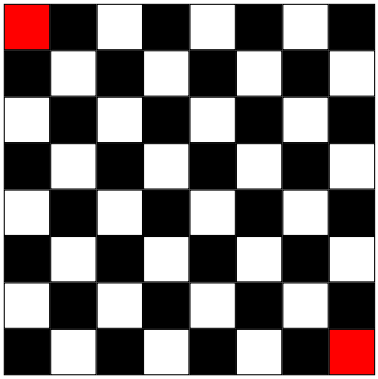
MathLovers

Solutions to the problems
from 08/21/2025

Problem 1. Determine in how many ways it is possible to tile with dominoes (of size 2×1) a standard chessboard (8×8) with the squares $A1$ and $H8$ removed.

Source of the problem: *"Critical thinking"*, Max Black, 1946
Problem selection and solution editing: Scarlett Lafa
Solution:



Zauważmy, że każda kostka domina położona na szachownicy zakrywa jedno białe i jedno czarne pole.
Natomiast liczba czarnych pól po usunięciu $A1$ i $H8$ wynosi 30, a białych ciągle 32, czyli jedna kostka domina musiałaby zakryć dwa białe pola – a to jest niemożliwe.
Notice that each domino placed on the chessboard covers one white and one black square.
Meanwhile, the number of black squares after removing $A1$ and $H8$ is 30, and the number of white squares is still 32, which means one domino would have to cover two white squares – and that is impossible.

Problem 2. The probability that:

- I'll eat chocolate today is 60%
- I'll eat chocolate tomorrow is 50%
- I won't eat chocolate either today or tomorrow is 20%

Calculate the probability that

a) I'll eat chocolate today or tomorrow

b) I'll eat chocolate today and tomorrow

Author of the problem: Scarlett Lafa
Solution: Let us denote the event of eating chocolate today as A, and the event of eating chocolate tomorrow as B. Therefore
$P(A) = 0.6$
$P(B) = 0.5$ $P(x)$ = probability of event x
and $P(A' \cap B') = 0.2$

a) The probability that I will eat chocolate today or tomorrow is $P(A \cup B) = 1 - P\left((A \cap B)'\right)$ from De Morgan's set-theoretic laws
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

we know that

$$P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.2 = 0.8$$

b) The probability that I will eat chocolate today and tomorrow is $P(A \cap B)$.
Notice that $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
Thus $P(A \cap B) = 0.6 + 0.5 - 0.8 = 0.3$