Problem 1. We are given five numbers:: a_1, a_2, a_3, a_4 i a_5. Their sum is equal to S. Furthermore, for each $k \in \{1, 2, \ldots, 5\}$ we have $a_k = k + S$. What is the value of S?

Source: *Mathematical Kangaroo* 2023 – category *Student*
Choice of problem: Maja Chlewicka
Solution: Let us write out the numbers according to the given formula:

$$\begin{cases} a_1 = 1 + S \\ a_2 = 2 + S \\ a_3 = 3 + S \\ a_4 = 4 + S \\ a_5 = 5 + S \end{cases}$$

Now, let us add them together, remembering that $a_1 + a_2 + a_3 + a_4 + a_5 = S$:

$$1 + S + 2 + S + 3 + S + 4 + S + 5 + S$$

$$15 + 5S = S$$

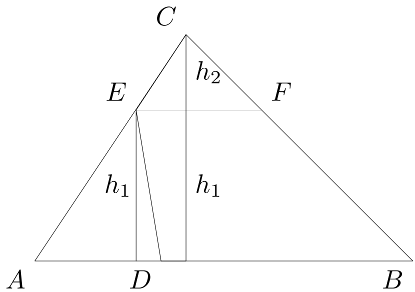
$$5S - S = -15$$

$$4S = -15$$

$$S = -\frac{15}{4}.$$

Problem 2. Given a triangle $\triangle ABC$. On side AB, a point D is chosen such that $AD : DB = 1 : 2$, and on side AC, a point E is chosen such that $AE : EC = 2 : 1$. Determine the ratio of the area of triangle $\triangle ADE$ to the area of quadrilateral $DBCE$.

Source: 2nd Regional Competition for primary school students of the Świętokrzyskie Voivodeship in mathematics – school year 2017/2018
Choice of problem: Maja Chlewicka
Solution:
Notice that triangle $\triangle ABC$ consists of triangle $\triangle ADE$ and quadrilateral $DBCE$, therefore:



$$P_{DBCE} = P_{ABC} - P_{ADE}$$

We are given:

$$\frac{|AD|}{|DB|} = \frac{1}{2}, \quad \frac{|AE|}{|EC|} = \frac{2}{1}$$

Then the area of triangle $\triangle ABC$ can be expressed as:

$$P_{ABC} = \frac{1}{2} \cdot |AB| \cdot (h_1 + h_2)$$

From the *angle-angle-angle* similarity, we observe that triangle $\triangle ABC$ and triangle $\triangle EFC$ are similar. Hence,

$$\frac{|AE|}{|EC|} = \frac{2}{1} \Rightarrow \frac{h_1}{h_2} = \frac{2}{1}$$

$$P_{ADE} = \frac{1}{2} \cdot |AD| \cdot h_1$$

$$P_{ADE} = \frac{1}{2} \cdot \frac{1}{3}|AB| \cdot \frac{2}{3}(h_1 + h_2)$$

$$P_{ADE} = \frac{2}{18} \cdot |AB| \cdot (h_1 + h_2)$$

$$P_{DBCE} = \frac{1}{2} \cdot |AB| \cdot (h_1 + h_2) - \frac{2}{18} \cdot |AB| \cdot (h_1 + h_2)$$

$$P_{DBCE} = \frac{7}{18} \cdot |AB| \cdot (h_1 + h_2)$$

$$\frac{P_{ADE}}{P_{DBCE}} = \frac{2}{7}.$$