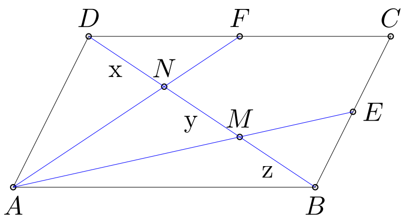
Solutions to the Problems
from 07/24/2025

**Problem 1.** Vertex $A$ of parallelogram $ABCD$ is connected by segments to the midpoints $E$ and $F$ of sides $BC$ and $CD$, respectively. These segments intersect the diagonal $BD$ at points $M$ and $N$. Prove that points $M$ and $N$ divide $BD$ into congruent segments.

**Source:** Ryszard Pagacz "Mała Olimpiada Matematyczna Zadania konkursowe z rozwiązaniami Tom 2. Geometria" wydawnictwo Oficyna Edukacyjna, wydanie I, 2023 r.
**Choice of problem:** Scarlett Lafa
**Solution:**




Let

$$|DN| = x, \ |NM| = y, \ |MB| = z.$$

The triangles $AMD$ and $BEM$ are similar (angle-angle). The similarity ratio is 2. Therefore:

$$x + y = 2z \tag{1}$$

The triangles $ABN$ and $FDN$ are also similar (angle-angle). The similarity ratio is also 2. Therefore:

$$y + z = 2x \tag{2}$$

Subtracting equation (2) from equation (1), we get:

$$x - z = 2z - 2x$$

$$x = z$$

Then from equation (1), we get $y = z$ so $x = y = z$.

**Problem 2.** Does there exist a triangle in which two of the sides have lengths 1 and 4, and two of the altitudes have lengths 3 and 4? Justify your answer.

**Source:** Ryszard Pagacz "Mała Olimpiada Matematyczna Zadania konkursowe z rozwiązaniami Tom 2. Geometria" wydawnictwo Oficyna Edukacyjna, wydanie I, 2023 r.
**Choice of problem:** Scarlett Lafa
**Solution:** Let $h_a$, $h_b$, and $h_c$ denote the lengths of the altitudes dropped onto the sides of lengths $a$, $b$, and $c$, respectively, with $a = 4$, $b = 1$. Then $h_c \leq b = 1$ and $h_a \leq b = 1$. This is a contradiction, because one of these altitudes is equal to 3 or 4. Therefore, such a triangle does not exist.