Problem 1. An urn contains white, blue, and black balls. We know that there are 6 more white balls than blue ones. Moreover, the probability of drawing a black ball in a single draw is $\frac{1}{2}$, and the probability of drawing a blue ball is $\frac{1}{7}$. How many balls of each color are there?

Problem author: Michał Fronczek
Solution: Let us denote the number of white, blue, and black balls by w, b, and k respectively. The total number of balls is $w + b + k$. The conditions given in the problem are as follows:

$$\begin{cases} w = b + 6 \\ \frac{k}{w+b+k} = \frac{1}{2} \\ \frac{b}{w+b+k} = \frac{1}{7} \end{cases}$$

From the second equation, multiplying both sides by the denominator gives us:

$$2k = w + b + k \quad \Rightarrow \quad k = w + b = (b + 6) + b = 2b + 6$$

Substituting into the third equation:

$$\frac{b}{w + b + k} = \frac{1}{7}$$

$$\frac{b}{(b + 6) + b + (2b + 6)} = \frac{1}{7}$$

$$\frac{b}{4b + 12} = \frac{1}{7}$$

Cross-multiplying:

$$7b = 4b + 12 \quad \Rightarrow \quad 3b = 12 \quad \Rightarrow \quad b = 4$$

Then:

$$w = b + 6 = 10, \quad k = w + b = 10 + 4 = 14$$

Therefore, there are 10 white, 4 blue, and 14 black balls.

Problem 2. Let a and b be two real numbers such that the numbers $a+b$, $ab\sqrt{a}$, and $ab\sqrt{b}$ are rational. Prove that a and b must also be rational.

Problem author: Michał Fronczek
Solution: First note that if either a or b is equal to 0, then from the assumptions we have that if $a=0$, then $a+b=b$ is rational, and similarly if $b=0$, then a is rational. Thus, in this case, the conclusion follows. Moreover, since the square roots are present in the expressions, we must have $a, b \geqslant 0$. So we now assume $a>0$ and $b>0$.

It is a fact that the product of rational numbers is also rational. In particular, the square of a rational number is rational. Therefore, the squares

$$(ab\sqrt{a})^2 = (ab)^2 \cdot a \quad \text{and} \quad (ab\sqrt{b})^2 = (ab)^2 \cdot b$$

must be rational.

Continuing, since the sum of rational numbers is rational, it follows that the number

$$(ab)^2 \cdot a + (ab)^2 \cdot b = (ab)^2(a+b)$$

is rational. But we know that $a+b$ is rational and nonzero, so $(ab)^2$ must also be rational. Moreover, it is nonzero.

Now consider the difference of two rational numbers, which must also be rational. So the number

$$(ab)^2 \cdot a - (ab)^2 \cdot b = (ab)^2(a-b)$$

is rational. Since $(ab)^2$ is rational and nonzero, it follows that $a-b$ is also rational.

Thus we now have that both $a+b$ and $a-b$ are rational. Therefore, their sum and difference:

$$2a = (a+b) + (a-b), \quad 2b = (a+b) - (a-b)$$

are also rational. Halving each gives that a and b must both be rational.

This completes the proof.