Solutions to the Problems
from 06/23/2025
(Father's Day)
You can expect the next set of problems later today!

Problem 1. In a table tennis tournament organized for Father's Day, 50 fathers and 50 sons took part. Each pair (consisting of one father and one son) played one match against every other pair, and there were no draws. Is it possible that every pair won the same number of matches? Justify your answer.

Problem source: 4th Junior Mathematical Olympiad
Problem selection and solution editing: Grzegorz Rudnicki
Solution: Such a situation is impossible.
Suppose that each pair won $k \in \mathbb{Z}$ matches. Then the total number of matches won—and therefore played—in the tournament is $50k$. On the other hand, each pair lost exactly $49 - k$ matches. Therefore, the total number of matches lost—and hence played—is $50(49 - k)$. Thus, $50k = 50(49 - k)$, which gives $k = \frac{49}{2} \notin \mathbb{Z}$. This contradiction completes the solution.