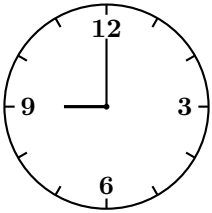
MathLovers

Problems – 06/19/2025

The solutions to the problems below
will be published on Sunday 06/22/2025

Problem 1. Imagine you are looking at an analog clock that shows 9:00. After how much time will the hour and minute hands form a 90° angle?



Problem 2. Solve the equation in real numbers:

$$x^{2025} + x^{2024}y + x^{2023}y^2 + \cdots + x^2y^{2023} + xy^{2024} + y^{2025} = 0.$$

Problem 3. ***Stereoisomers*** are molecules that have the same chemical composition (i.e. they consist of the same atoms in the same quantities) and the same connectivity between those atoms, but they differ in the three-dimensional arrangement of the atoms.

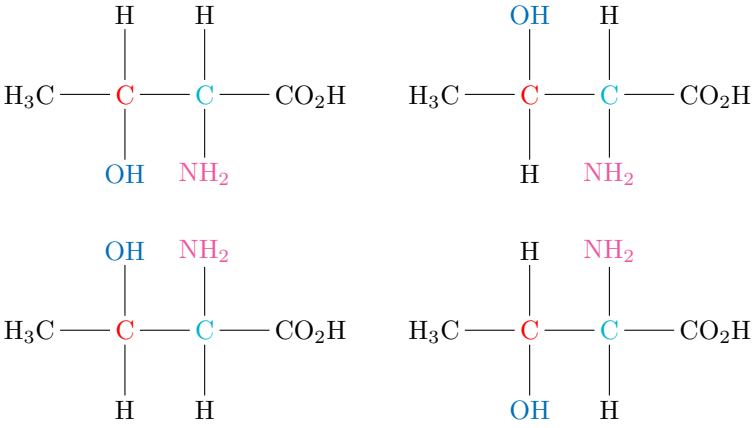
Think of it like this: you have two LEGO figures built from exactly the same bricks. Every brick is identical and appears the same number of times. Yet one figure has its arm stretched to the right, the other to the left. They are "the same", but they look different – and can behave differently, for example fitting with other pieces in different ways.

In chemistry, stereoisomers can also behave differently – even though they share the same "recipe" (molecular formula), their different spatial arrangements give rise to different properties such as taste, smell, or biological activity.

In organic chemistry, each carbon is represented with four bonds. Below is a certain compound: 2-amino-3-hydroxybutanoic acid (the exact name is not important). It has *four* stereoisomers (assume mirror images count as distinct stereoisomers). Treat a "group" as a chain of atoms connected by bonds, which

for this problem we can regard as a single atom.

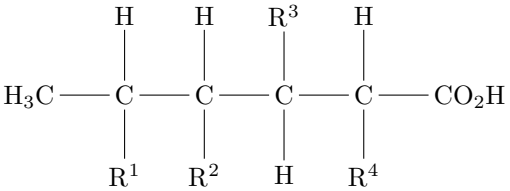
At the red carbon C, the OH group can be above or below; at the blue carbon C, the NH_2 group – likewise.



How many stereoisomers does a hypothetical compound have in which between the groups

$$H_3C - \ldots - CO_2H$$

there are n carbons, and from each carbon i there departs exactly one hydrogen H and one group R^i, where throughout the whole molecule the groups R^1,R^2,...,R^n are pairwise distinct and different from H, H_3C, and CO_2H? For a clearer picture, the compound for $n = 4$ is drawn below:



Problem 4. For which positive integers $n \in \mathbb{Z}_+$ does the expression

$$\frac{4n^4 - 48n^3 + 168n^2 - 144n + 60}{n^2 - 6n + 3}$$

take positive integer values?

Good luck!