Problem 1. Square $ABCD$ has side length 3. Let E be a point on segment CD such that $DE = 1$. Segments AE and BD intersect at point F. Let D', E' be the reflections of points D, E with respect to F, respectively. Find the area of quadrilateral $ABD'E'$.

Problem author: Bartosz Trojanowski
Solution: We denote the area of figure X by $[X]$. Triangles DEF and $D'E'F$ are congruent by the side-angle-side criterion, so they have equal areas. It follows that

$$[ABD'E'] = [ABCD] - ([ADF] + [D'E'F]) - [BCD] = [ABCD] - [ADE] - [BCD].$$

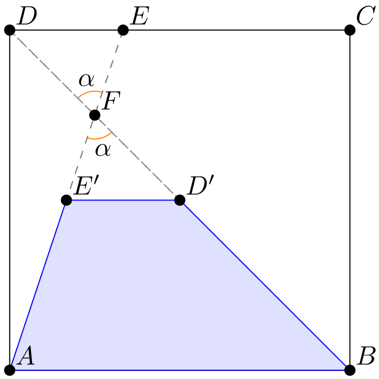
We can easily compute:

$$[ABCD] = 3 \cdot 3 = 9,$$
$$[ADE] = \frac{1 \cdot 3}{2} = \frac{3}{2},$$
$$[BCD] = \frac{3 \cdot 3}{2} = \frac{9}{2},$$

so

$$[ABD'E'] = 9 - \frac{3}{2} - \frac{9}{2} = 3.$$

Thus, the sought area is 3.



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Problem 2. Find all triples of positive real numbers a, b, c that satisfy the system of equations

$$\begin{cases} a = bc^2 \\ b = ca^2 \\ c = ab^2 \end{cases}$$

Problem author: Bartosz Trojanowski

Solution: Since a, b, c are positive, they are not equal to zero. Multiplying the three equations side by side, we get $abc = 1$. Hence,

$$a = bc^2 = \frac{c}{a} \iff c = a^2.$$

We perform similar operations on the second and third equations, and eventually obtain the system:

$$\begin{cases} a = b^2 \\ b = c^2 \\ c = a^2 \end{cases}$$

So we have

$$a = b^2 = c^4 = a^8 \implies a^7 = 1.$$

Similarly, we find $b^7 = 1 = c^7$, which means the only solution to the system is the triple $a = b = c = 1$.

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