MathLovers

Problems 06/16/2025

Solutions to the following problems
will be published on Thursday 06/19/2025.

Problem 1. Square $ABCD$ has side length 3. Let E be a point on segment CD such that $DE = 1$. Segments AE and BD intersect at point F. Let D' and E' be the reflections of points D and E with respect to point F, respectively. Find the area of quadrilateral $ABD'E'$.

Problem 2. Find all triples of positive real numbers a, b, c satisfying the system of equations:

$$\begin{cases} a = bc^2 \\ b = ca^2 \\ c = ab^2 \end{cases}$$

Good luck!

Note: Hints for the problems can be found on the next page.

Problem 1:

Hint 1: Notice that triangles DEF and $D'E'F$ have equal areas (why?).

Hint 2: It follows that $[ADE] = [ADF] + [D'E'F]$, where $[XYZ]$ denotes the area of triangle XYZ.

Problem 2:

Hint 1: Multiply all equations together.

Hint 2: The given system of equations is equivalent to:

$$\begin{cases} a = b^2 \\ b = c^2 \\ c = a^2 \end{cases}$$

(why?).