Problem 1. Do there exist positive integers a, b, c such that each of the numbers ab, bc, ca ends with the digits 20?

Source: *Kwadrat* no. 11, Journal of the Junior Mathematical Olympiad, December 2013
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Solution: Note that if a number ends in the digits 20, then it is divisible by 5 but not by 25 (since numbers divisible by 25 end in 00, 25, 50, or 75). Suppose such integers a, b, c exist as described in the problem. Then each of the numbers ab, bc, ca has exactly one factor of 5 in its prime factorization. Hence the product $ab \cdot bc \cdot ca = (abc)^2$ has exactly three factors of 5 in its prime factorization, which is impossible because it is a perfect square (in a square, all prime factors appear an even number of times).

Problem 2. Find the remainder when 345^{679} is divided by 16.

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Solution: Observe that

$$345^{679} \equiv 9^{679} \pmod{16}$$

We know that

$$9 \equiv 9 \pmod{16}$$

and

$$9^2 \equiv 1 \pmod{16}.$$

So we can write:

$$345^{679} \equiv (9^2)^{339} \cdot 9 \equiv 1 \pmod{16}.$$

Thus, the sought remainder is 1.