# Zadania

**Zadanie 1.** Find all positive integers $a$ and $b$ such that the numbers $ab+1$, $2a+b$ and $a+(3b)^2+1$ are prime.

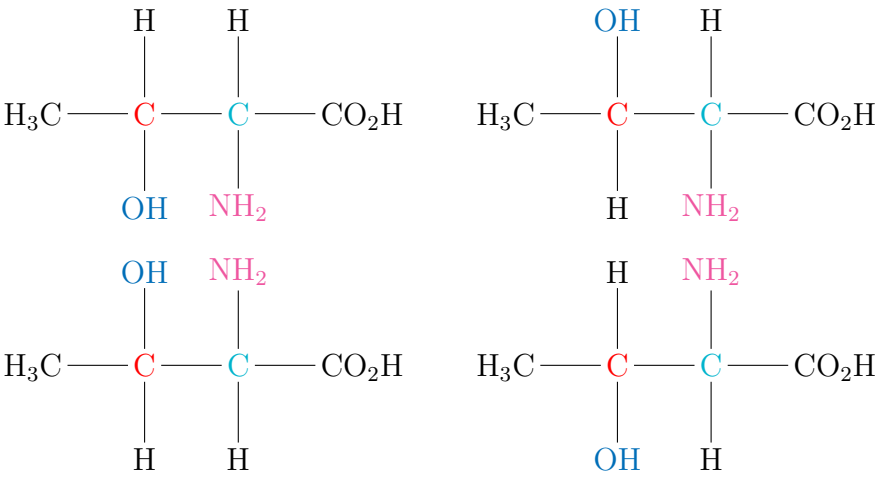
**Zadanie 2.** An isosceles triangle $ABC$ is given, whose base $AB$ is 12 cm long. The base angles measure 30°. Calculate the area of this triangle.

**Zadanie 3.** A positive integer $n$ is called a *meowmeowhammer* if there exist positive integers $a, b$ satisfying

$$n = ab + a + b.$$

Determine all positive integers $n$ such that the number $n$ is a *meowmeowhammer* and the number $n+1$ is prime.

**Zadanie 4.** ***Stereoisomers*** are molecules that have the same chemical composition (i.e., they consist of the same atoms and have the same number of each type of atom), have the same connections between atoms, but differ in the spatial arrangement of those atoms.
Imagine it like this: you have two LEGO figures built from exactly the same bricks. Each brick is identical and appears in the same quantity. But one figure has its arm stretched to the right, while the other to the left. Seemingly the same, but they look different – and they can behave differently, for example, fitting with other elements in different ways.
In chemistry, stereoisomers can also behave differently – although they have the same "recipe" (chemical formula), due to different arrangements of atoms they can have different properties: taste, smell, or even biological activity.
In organic chemistry, each carbon atom is drawn with 4 bonds. Below, a certain chemical compound is shown. It is 2-amino-3-hydroxybutanoic acid (the name is not important). Notice that it has 4 stereoisomers (let us assume that mirror images count as different stereoisomers). A group is a chain of atoms connected by bonds, which for the purposes of this problem can be treated as a single atom. At the red carbon C, the group OH can be placed either at the top or at the bottom. At the turquoise carbon C, the group $NH_2$ behaves the same way.

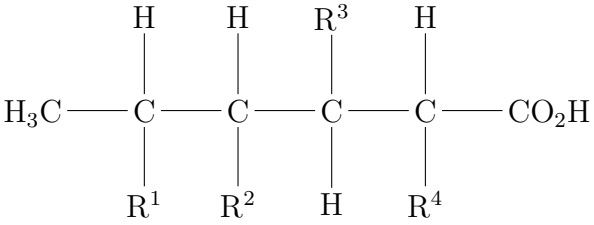



How many stereoisomers does a hypothetical compound have, which between the groups

$$H_3C - \ldots - CO_2H$$

has $n$ carbons, where from each carbon $i$ exactly one hydrogen H and one group $R^i$ depart, where throughout the molecule the groups $R^1$, $R^2$, ..., $R^n$ are pairwise different and distinct from H, $H_3C$ and $CO_2H$?

For better illustration, the compound for $n = 4$ is shown below:



# Rozwiązania

**Zadanie 1.** Find all positive integers $a$ and $b$ such that the numbers $ab+1$, $2a+b$ and $a+(3b)^2+1$ are prime.

**Autor zadania:** Michał Fronczek
**Rozwiązanie:** Let us note that $2a+b \geqslant 2\cdot 1+1=3$, and since it must be a prime number, it has to be odd. However, notice that $2a$ is even regardless of $a$, so for the whole sum to be odd, $b$ must be odd. Similarly, $a+(3b)^2+1 \geqslant 1+(3\cdot 1)^2+1=11$ must also be odd, and from the oddness of $b$, hence also of $3b$ and finally $(3b)^2$, it follows that $a$ must be odd. Now observe that in this case $ab+1$ is even. The only even prime number is 2, so $ab+1=2$, which means $ab=1$. We know that $a$ and $b$ are positive integers, and therefore the only possibility is $a=b=1$.
It remains to check whether the obtained numbers are indeed a solution. We have:
$ab+1=1\cdot 1+1=2$ works
$2a+b=2\cdot 1+1=3$ works
$a+(3b)^2+1=1+(3\cdot 1)^2+1=1+9+1=11$ also works
Hence, the only valid solution is $a=b=1$.

**Odpowiedź:** $a=b=1$

**Zadanie 2.** A isosceles triangle $ABC$ is given, whose base $AB$ is 12 cm long. The base angles measure 30°. Calculate the area of this triangle.

**Autor zadania:** Maja Chlewicka
**Rozwiązanie:** We draw the height $CD$ from vertex $C$ dropping onto the midpoint of the base $AB$. This creates two triangles – $ADC$ and $DBC$ – with angles measuring 30°, 60°, and 90°. Using triangle $DBC$ as an example, we calculate the length of the height $CD$ based on the properties of a triangle with angles 30°, 60°, and 90°.
$DB$ measures 6 cm because it is half of the base.

$$\frac{\sqrt{3}}{2}a = 6$$

$$a = 4\sqrt{3}$$

$$CD = \frac{1}{2}a$$

$$CD = 2\sqrt{3}$$

$$P = \frac{1}{2}\cdot 2\sqrt{3}\cdot 12 = 12\sqrt{3}\ \text{cm}^2.$$

**Odpowiedź:** 12 cm$^2$.

**Zadanie 3.** A positive integer $n$ is called a *meowmeowhammer* if there exist positive integers $a, b$ satisfying

$$n = ab + a + b.$$

Determine all positive integers $n$ such that the number $n$ is a meowmeowhammer and the number $n+1$ is prime.

**Autor zadania:** Bartosz Trojanowski

**Rozwiązanie:** Suppose that the number $n$ is meowmeowhammer. Note that

$$n+1 = (ab+a+b)+1 = (ab+a)+(b+1) = a(b+1)+(b+1) = (a+1)(b+1).$$

The numbers $a+1$ and $b+1$ are greater than 1, so if the number $n$ is meowmeowhammer, then the number $n+1$ is composite. Therefore, such an $n$ does not exist. ■

**Zadanie 4.** ***Stereoisomers*** are molecules that have the same chemical composition (i.e., they consist of the same atoms and have the same number of each type of atom), have the same connections between atoms, but differ in the spatial arrangement of those atoms.
Imagine it like this: you have two LEGO figures built from exactly the same bricks. Each brick is identical and appears in the same quantity. But one figure has its arm stretched to the right, while the other to the left. Seemingly the same, but they look different – and they can behave differently, for example, fitting with other elements in different ways.
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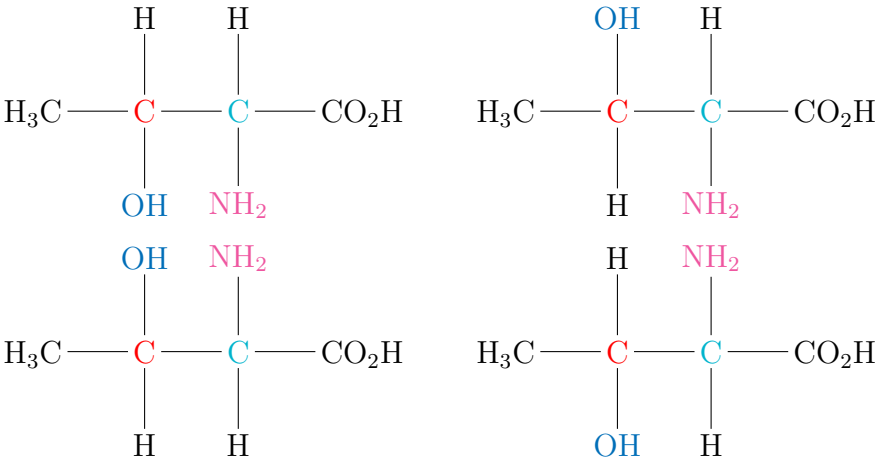

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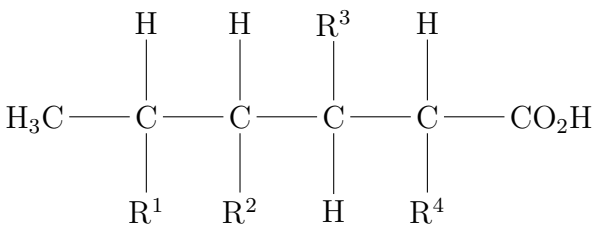

**Autor zadania:** Tomasz Kossakowski

**Rozwiązanie:** Let us notice that for the placement of each group $\mathrm{R}^i$ at the $i$-th carbon atom, we have two possibilities — top or bottom. Therefore, by the rule of multiplication, the total number of stereoisomers is

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{n \text{ twos, since there are } n \text{ carbon atoms}} = 2^n.$$

■

# I Kielecki Dwujęzyczny Konkurs Matematyczny

**Zadanie przykładowe**

Przedstawione zostały pewne liczby w języku chińskim[1]:

| | | | |
|---|---|---|---|
| 二 | 2 | 二十六 | 26 |
| 五 | 5 | 五十五 | 55 |
| 七 | 7 | 四十一 | 41 |
| 九 | 9 | 八十 | 80 |
| 一十一 | 11 | 一百八十二 | 182 |
| 一十六 | 16 | 五百三十九 | 539 |
| 二十三 | 23 | 四千二百六十八 | 4268 |

Jak słownie w języku chińskim zapiszemy wynik poniższego działania?

四千八百二十八 + 二千三百三十二

[1]Język chiński jest jednym z najstarszych i najbogatszych języków świata, używanym przez ponad miliard ludzi, głównie w Chinach i na Tajwanie. Charakteryzuje się unikalnym systemem pisma opartym na znakach (汉字), które łączą znaczenie i wymowę. W mowie istnieje wiele dialektów, z których najważniejszy i oficjalny to mandaryński (普通话). Chiński jest językiem tonalnym – znaczenie wyrazu zależy od tonu, w jakim zostanie wypowiedziany. Dzięki swojej długiej historii i wpływowi kulturowemu odgrywa ogromną rolę nie tylko w Azji, lecz także na całym świecie.

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**Autor zadania:** Tomasz Kossakowski
**Rozwiązanie:** Struktura to:

$$X \text{ 千 } Y \text{ 百 } Z \text{ 十 } W = 1000X + 100Y + 10Z + W$$

gdzie $X, Y, Z, W$ to cyfry jak poniżej:

| | | | |
|---|---|---|---|
| 一 | 1 | 六 | 6 |
| 二 | 2 | 七 | 7 |
| 三 | 3 | 八 | 8 |
| 四 | 4 | 九 | 9 |
| 五 | 5 | | |

Zatem 四千八百二十八 + 二千三百三十二 to $4828 + 2332 = 7160$, czyli po chińsku 七千一百六十。

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