Zadania

Zadanie 1. Given integers a, b, c and d such that $a+b+c+d=1$. Show that

$$\min\left(a^2+b-cd,\ c+d-b^2,\ b^2-a^2,\ a+b,\ cd-b\right) \leqslant \frac{1}{5}.$$

Note: Function $\min(a_1, a_2, \ldots, a_n)$ takes the value of the smallest argument.

Zadanie 2. Let's call a positive integer n an *extramultitrain choo-choo number*, if there exist positive pairwise distinct divisors d_1, d_2, d_3, of the number n satisfying

$$d_1 + d_2 + d_3 = n.$$

Prove that there are infinitely many *extramultitrain choo-choo numbers*.

Rozwiązania

Zadanie 1. Given integers a, b, c and d such that $a+b+c+d=1$. Show that

$$\min\left(a^2+b-cd,\ c+d-b^2,\ b^2-a^2,\ a+b,\ cd-b\right) \leqslant \frac{1}{5}.$$

Autor zadania: Michał Fronczek
Rozwiązanie: It is a fact that the minimum of a certain set of numbers is not less than the average of all the elements of this set. Therefore:

$$\begin{aligned}\min\left(a^2+b-cd,\ c+d-b^2,\ b^2-a^2,\ a+b,\ cd-b\right) &\leqslant \\ \leqslant \frac{(a^2+b-cd)+(c+d-b^2)+(b^2+a^2)+(a+b)+(cd-b)}{5} &= \\ = \frac{a^2+b-cd+c+d-b^2+b^2+a^2+a+b+cd-b}{5} = \frac{a+b+c+d}{5} &= \frac{1}{5}.\end{aligned}$$

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$$d_1+d_2+d_3=n.$$

Prove that there are infinitely many *extramultitrain choo-choo numbers*.

Autor zadania: Bartosz Trojanowski
Rozwiązanie: We will prove that every number divisible by 6 is a *extramultitrain choo-choo number*. Indeed, if $6 \mid n$, then the numbers $\frac{n}{2}, \frac{n}{3}, \frac{n}{6}$ are pairwise distinct and integers, and also

$$\frac{n}{2}+\frac{n}{3}+\frac{n}{6} = n\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right) = n,$$

which completes the proof. ■